Games, graphs, and machines



October 18, 2024

"Nim-sum" for all games: Grundy value

$$mex(0,1,2,4) = 3$$

- 1. All sink states get 0
- 2. Each state gets mex of its children.

"Nim-sum" for all games: Grundy value



Another example

 $2\times 2\ Chomp$



Key properties



Nim(3)Champ (2,2) :----+ Moile 1 $(10)_{2}$ O Nim(2)Onale 2: (11)₂ ~ Unavailable.

Key properties

- 1. Grundy value zero \iff P
- 2. Grundy value non-zero $\iff N$
- 3. Grundy value of G + H = Grundy value of $G \oplus$ Grundy value of H.

Grundy value of Chomp(2, 2) + Nim(3, 4)

Theorem: Two games are stably equivalent if and only if they have the same Grundy value.